The Effect of a Superposed Constant Field Upon The Alternating Current Permeability and Energy Loss In Iron

DISSERTATION

PRESENTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN THE GRADUATE SCHOOL OF THE OHIO STATE UNIVERSITY

BY
ALVA W. SMITH

The Ohio State University
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INTRODUCTION

Investigations on the effect of a superposed constant induction upon small cyclic changes in a magnetic substance have been carried out by Rayleigh[1], Ewing[2], Gans[3], Madelung[4], Hoffman[5], Campbell[6] and others. Much of the systematic work in this field has been done by producing small cyclic changes by sudden increments and decrements to the existing constant field. It appeared that further study might profitably be made with alternating current to see to what extent the constant field changes the permeability to a rapidly alternating field and whether the previous history influences this permeability and the resulting energy losses in iron.

Aside from its bearing upon theories of magnetism, this problem is of interest from a technical point of view. In the double frequency transformer, the telephone receiver and in numerous other applications of the polarized electromagnet, the direct current field has superposed upon it an alternating field. The performance of these devices is conditioned by the change in effective permeability to an alternating field when the core is under the magnetizing action of this constant longitudinal field.

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1. Rayleigh, Phil. Mag. 23, 1887, p. 225; 38, 1894, p. 295.
2. Ewing & Klaassen, Phil. Tr. A, 184, 1893, p. 1030.
3. Gans, Ann. der Phys. 27, 1908; 29, 1909; 33, 1910. Phys. Zeitsch. 1911; Ann. der Phys. 61, 1920.
4. Madelung, Phys. Zeitsch, 1912, p. 436.
5. Hoffman, Archiv. f. Electrotechnik, 1913, p. 433.
6. Campbell, Phys. Soc. of London Proc. June, 1920.

METHOD OF MEASUREMENT

The ratio of the apparent inductance of an iron-cored toroid to the inductance of the same coil with an air core is taken as the measure of the alternating current permeability. The inductance which the coil would have with an air core was calculated from the number of turns and dimensions of the coil. Nearly simultaneous measurements were made of the apparent inductance and resistance of the iron cored coil. The difference between the ap-

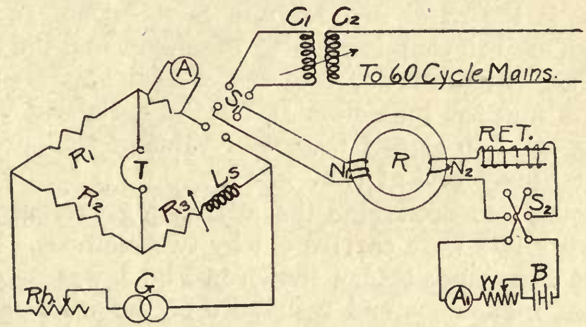

FIG. 1

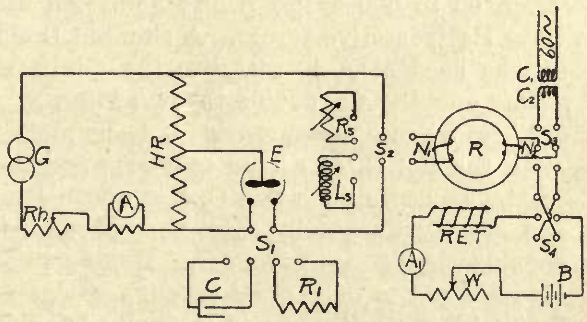

FIG. 2

parent resistance measured with an alternating current and the true resistance measured with direct current is the increase in resistance due to core losses. This increase in resistance multiplied by the square of the effective alternating current gives the core loss due to the combined action of hysteresis and eddy currents. No attempt was made to separate these losses.

The ordinary or normal magnetization curve and the hysteresis loops were determined by ballistic measurements. By means of a direct current in winding N_2 (Figs. 1 and 2), the iron core was brought into a state represented by a given point on one of these static characteristics, and measurements were then made of the apparent inductance and resistance of winding N_1. A series of such readings was made at various points on these curves.

Field intensities were calculated by the use of a formula, due to Kirchhoff, for a uniformly wound ring of n turns uniformly magnetized:

$$H = \frac{0.2nI}{a}\left[\frac{a}{R} + \frac{1}{12}\left(\frac{a}{R}\right)^3 + \frac{1}{80}\left(\frac{a}{R}\right)^5 + \text{—}\right] \text{ gauss,} \quad (1)$$

in which a is the axial breadth and R the mean radius. This formula was used in computing both the direct and the alternating current fields. In the latter case the current was assumed to be of sine form and the maximum H for the cycle was obtained by substituting for I in Eqn. I the meter value of the current multiplied by the square root of two.

Measurements to determine the alternating current permeability and energy loss were carried out by two methods. In the first method, the impedance bridge shown in Fig. 1 was used to obtain the apparent inductance and resistance of the winding N_1 of the ring R. For G three different generators were used, giving frequencies from 60 to 600 cycles per second. All of these gave nearly sine wave electromotive forces. A shunted Duddell thermogalvanometer was used at A to measure the alternating current through N_1. The double-pole, double throw switch S_1 connects N_1 into the bridge arm when a measurement is to be made or connects it in series with the coil C_1 of a transformer when the ring is to be demagnetized. To accomplish this C_2 is connected to the laboratory mains and is placed coaxially upon C_1. Then C_2 is gradually moved to a considerabe distance from C_1. During this removal of C_2, the iron is assumed to be carried through hysteresis cycles of constantly diminishing amplitude and is finally left demagnetized.

The direct current to produce the superposed constant field is supplied to the terminals of N_2 by a storage battery B, connected through a regulating rheostat W, an ammeter A_1, a reversing switch S_2 and several retardation coils RET. This arrangement makes it possible to bring the iron into a known state of magnetization corresponding to points on the previously determined normal curve or on a hysteresis loop. The bridge measurement is then made with a small alternating current, the effective value of which is kept constant for a series of direct current fields. The retardation coils so completely annul the alternating current in the direct current circuit that its reaction does not reduce the primary inductance by more than one percent.

At the lower frequencies where the telephone is no longer sufficiently sensitive in making bridge balances, the electrometer method of measuring inductance previously described by the writer (Phys. Rev. Oct. 1919) was used. In this method, provision is made for measuring the power loss in N_1 (Fig. 2), as well as the inductance. The needle of the electrometer E is connected to the middle of the high resistance HR, the halves of which are two exactly similar coils of 50,000 ohms resistance.

When an inductance measurement is to be made, the quadrants of the electrometer are connected across the capacity C, and N_1 is connected into the gap S_2. The alternating current registered by A is regulated by means of the rheostat Rh to the desired value and the electrometer deflection observed. The inductance standard L_s is then substituted for N_1 in the gap S_2, the current regulated to its previous value and the electrometer deflection again read. The two inductances are directly proportional to their respective deflections. In other respects, this method is like the bridge method.

To measure the power loss in N_1, the electrometer quadrants are connected across the non-inductive resistance R_1, and N_1 is connected into the gap S_2. The alternating current is regulated to the desired value and the electrometer deflection read. Then the known resistance R_s is substituted for N_1, the current regulated to its previous value and the electrometer deflection again read. The ratio of the effective resistance of N_1 to the resistance of R_s is equal to the ratio of these respective deflections[1].

The bridge method is the more convenient for the smaller currents and higher frequencies, where its sensibility far exceeds that of the second method. On the other hand, the lack of sensitivity at lower frequencies gives the relative advantage to the electrometer method, particularly for the larger currents, for the deflections are proportional to the square of the current, other factors being the same.

Each measured value of inductance was divided by the inductance of the same toroid with an air core to obtain the permeability of the iron. Two methods of calculating this inductance were used. In the first of these, the following equation was used:

$$L = \frac{n\ A\ H}{10^8\ I} \text{ henrys} \qquad (2)$$

[1] F. M. Laws, Elect. Measurements, p. 322.

A is the sectional area of the iron core, n is the number of turns in N_1, and $\frac{H}{I}$ is obtained from Eqn. 1. The second formula used in calculating the inductance is taken from the Bulletin of the Bureau of Standards, Vol. S, No. 1. It is—

$$L = 2\, n^2\, a \log \frac{r_2}{r_1} \text{ henrys} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

In this formula, a is the axial thickness and r_2 and r_1 are respectively the outer and the inner radii of the ring. The difference between the values calculated by the two methods was of the order of one-fourth of one percent. The calculated values are recorded in Table I.

TEST PIECES

Three different specimens of iron showing widely different magnetic properties were studied. Each test piece was in the form of a ring. Ring I was built up from stampings of transformer plate of 16 mils thickness. The plates were given a thin layer of shellac, dried and assembled. A single layer of tape was applied to hold the plates together and over this were wound two windings of a single layer each of double silk covered No. 21 magnet wire. Ring II was similarly prepared except that dynamo plate was used. Ring III was made by placing two dust cores with their faced surfaces together. These cores are prepared by treating powdered electrolytic iron with shellac and compressing into the form of rings[1].

TABLE I

Ring	No. of Plates	Radial Thickness in CM.	Axial Thickness in CM.	Mean Diam. in CM.	Turns in N_1	Turns in N_2	Induct of N_1 Milhenry
I	54	0.952	2.255	14.11	473	437	0.136
II	28	0.635	0.995	9.29	302	293	0.247
III	2	2.15	0.535	7.55	190	183	0.0458

[1] Speed and Elmen, Jour. A. I. E. E., July, 1921.

Static Curves. In Fig. 3, the normal magnetization curves and hysteresis loops for Rings I and II are given. The permeability of I rises to a maximum of 4200 at H = 1.3 gauss; that of II to a maximum of 1430 at H = 5.1 gausses. The remanent magnetism is nearly the same for both, but the coercive force for Ring II is more than twice as great as for Ring I. For a range of B max. from 7400 to 12000, the hysteresis loss for Ring I obeys the parabolic law, and has a Steinmetz exponent of 2.13; but for flux densities above 12000, the losses appear to increase faster than

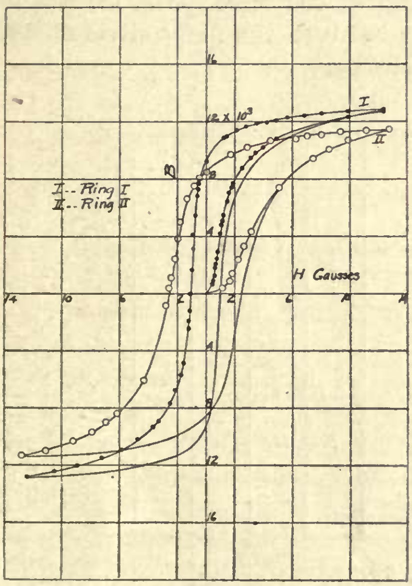

FIG. 3

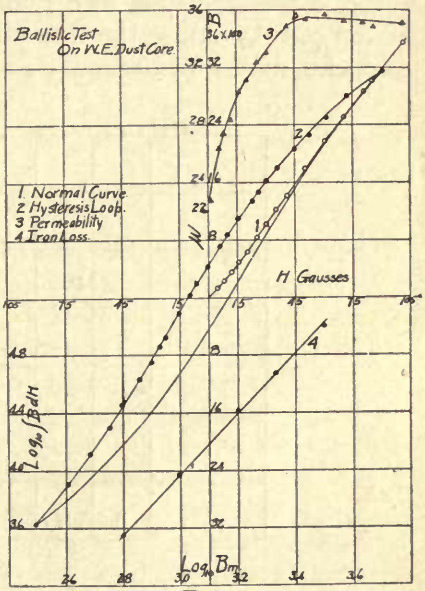

FIG. 4

would be predicted by this law. Ring II gives a value of 1.69 for this exponent.

The magnetic characteristics of the dust core, (Fig. 4), are strikingly different from those of the first two test rings. The normal curve shows an inclination to bend at low fields like any other sample of iron. But from 60 gausses up to 101 gausses, the upper limit studied, the B-H curve is nearly linear. The permeability rises from 22.6 at a field intensity of one gauss to 35.5 at fifty gausses, beyond which it remains nearly constant. Hysteresis loops for this specimen, with values of B max. from 630 to 3200, show that the remanent induction for each loop is from 8 to 10 per cent of its maximum induction. For the smallest loop, the coercive force is 10 per cent of the maximum field intensity; but for each of the other loops for B max. greater than 900, the coercive force is uniformly 13 per cent of the maximum field intensity.

In the lower half of Fig. 4, the logarithms of the areas of these loops are plotted against the logarithms of the maximum and inductions. From this graph the following facts can be deduced: (1) the hysteresis loss for these dust cores obeys the parabolic law; (2) the Steinmetz exponent of Bm is 2.09; (3) the coefficient is 0.0005.

The relatively low and nearly constant permeability of these dust cores is probably due to a reduction of the effective cross section of the core by filling the interstices between the finely divided iron particles with a non-magnetic substance.

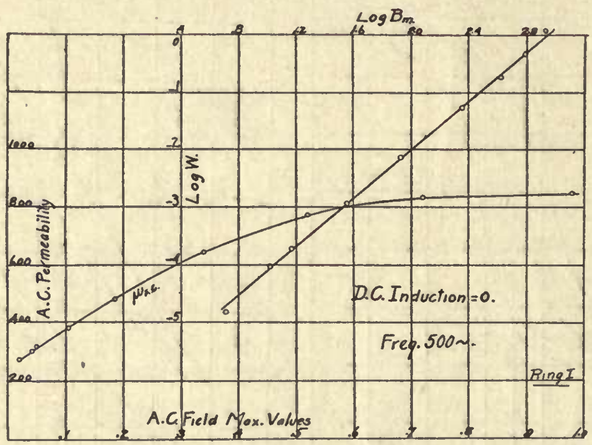

FIG. 5

Permeabilities and Losses with no Superposed Direct Current Field. The results obtained with Ring I demagnetized and with no direct current induction are given in Fig. 5. From an initial value of 250, the alternating current permeability rises to what appears to be a maximum at a value of about one-fifth of the maximum direct current permeability.

Each maximum value of the alternating field is multiplied by the corresponding value of the permeability to obtain the maximum value of the induction. The logarithms of these inductions are plotted in the insert in Fig. 5 against the logarithms of the measured iron-losses at these inductions. The equation of the curve is of the form $W = k B_m^a$.

From this curve $a = 2.13$. From a similar curve for Ring II, for H from .025 to 3.5 gausses, $a = 2.35$. The alternating current permeability for Ring II rises from an initial value of 100 to a value of 525 at H = 3.5 without indications of a maximum.

For Ring I, the exponent of B_m in the above equation is near to the value 2.16, which is the exponent of B_m from ballistic measurements. For Ring II the exponent for the total loss is 2.35; for the hysteresis loss alone it is 1.69 from the static curves. It was not expected that these two exponents should be the same for the reasons that they cover a different range of inductions; the alternating current loss includes eddy current losses; and the actual shapes of the hysteresis loops for very slow cycles is very different from those for very rapid cycles, as was shown by Ewing.[1]

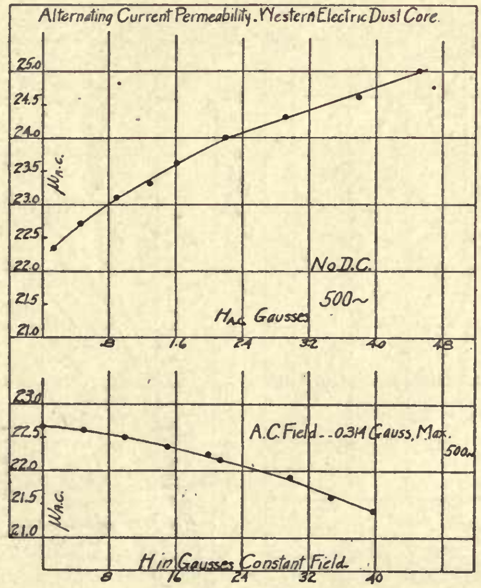

FIG. 6

In the upper part of Fig. 6, the alternating permeability of the dust core at 500 cycles per second with no superposed field is given as a function of the maximum value of the alternating field. The initial value of the permeability is 22.1 and the value reached at 4.5 gausses is 25.0. Ballistic tests give a value of 22.6 for the initial permeability and data supplied by the Western Electric Co. give 22.1 for its value at H = 1 gauss and 1000 cycles per second. The very small change of permeability with frequency can be attributed to a rather complete elimination of eddy currents on the one hand and on the other to the introduction of a substance with nearly constant reductivity between the particles of iron.

[1] Loc. cit.

The Effect of a Superposed Constant Induction. The dependence of the alternating permeability upon the direct current induction for Rings I and II is shown by the lower groups of curves of Figs. 7 and 8 respectively. The permeabilities are plotted as ordinates and the superposed inductions on the normal magnetization curve as abscissae. The permeability falls rapidly with small direct current inductions to a certain point after which the decrease is more gradual. The abrupt change in slope appears to come where the direct current field has increased to such a point that the resultant of the direct current and alternating current fields no longer assumes negative values, or in other words where the field ceases to be alternating and becomes pulsating.

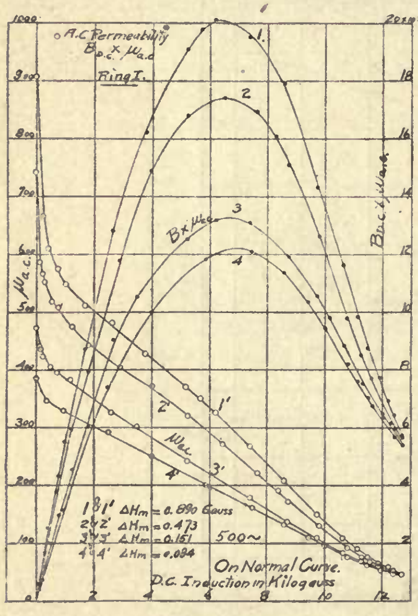

FIG. 7

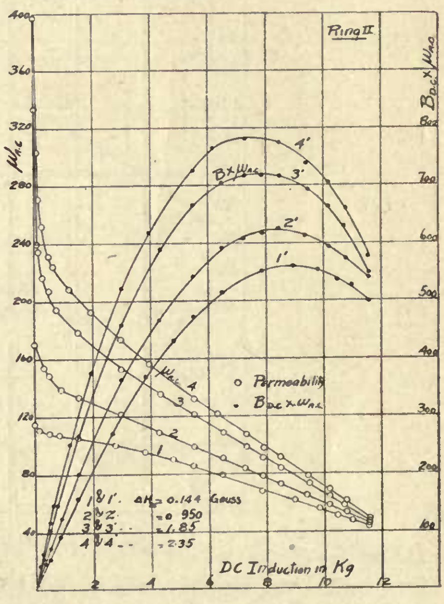

FIG. 8

For different values of alternating field, the permeabilities differ most from each other when there is no direct current induction. With increasing direct current inductions these curves converge. Beyond the point of convergence the permeability is practically independent of the alternating current induction. Campbell[1] has recently published data showing a rise of permeability with small superposed direct current fields. I have not observed this rise in any of my samples.

From the lower part of Fig. 6 one sees that the superposed direct field has much less influence upon the alternating permeability

[1] Loc. cit.

of the dust core than upon the other specimens. With a direct field of forty gausses, its permeability has decreased only 5 per cent of its original value; with a field of only twelve gausses that of Ring I has decreased from 90 to 95 per cent of its original value with no superposed field.

A typical set of data for Ring I is given in Table II. These data are taken at points on the normal magnetization curve and are plotted in Fig. 7, curve 4.

TABLE II

Ha.c. = .094		Freq. = 500 cycles per sec.	
Hd.c. gauss	μ a.c.	Bd.c. x μa.c.	Energy loss microwatts
0	386	0	1695
0.37	344	120	1070
0.56	329	313	1000
0.99	249	995	650
1.43	199	1175	382
2.48	132	1135	146
3.84	94	940	78.5
5.82	68	760	39.3
7.45	57.5	675	27.2
10.46	45.0	558	15.1

The Force Function. In his discussion of the telephone receiver Rayleigh makes use of Maxwell's equation for the force action of an electro-magnet upon its armature. Assume that this force per unit area of pole face is given by

$$F = K\ B^2 \quad \text{(4)}$$

If we consider the induction B as due to a constant component B_o and an alternating induction $B_m \sin \omega t$ produced by a current $I_m \sin \omega t$, we can write

$$F = K\ (B_o^2 + 2B_oB_m \sin \omega t + \frac{B_m^2}{2} - \frac{B_m^2}{2} \cos 2\omega t) \quad \text{(5)}$$

From this equation it appears that the force consists of three components, one a constant force proportional to $(B_o^2 + \frac{B_m^2}{2})$; a second,

$$F_2 = 2KB_oB_m \sin \omega t \quad \text{(6)}$$

of the same frequency as the current; and a third

$$F_3 = \frac{K}{2} B_m^2 \cos 2\omega t \qquad (7)$$

of double the frequency of the current.

If in Eqn. (6), for B_m we substitute

$$B_m = H_m \mu a.c. = K_2 I_m \mu a.c. \qquad (8)$$

in which K_2 is a constant, we get

$$F_2 = 2KK_2 \; B_o \mu a.c. \; I_m \sin \omega t \qquad (9)$$

The change in force per unit current is therefore

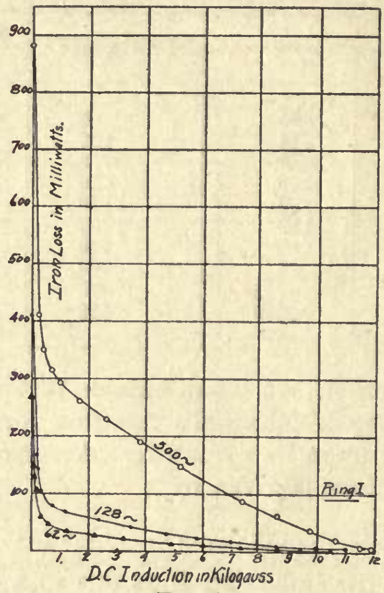

FIG. 9

$$\frac{dF_2}{dI_m} = 2KK_2 \; B_o \mu a.c. \sin \omega t \qquad (10)$$

The quantity $\frac{dF_2}{dI_m}$ is a measure of the force sensitivity of the polarized electromagnet. Equation (10) states that the change in pull per unit current of the *same frequency* is proportional to the product $B_o x \mu a.c.$ To make the response of the same frequency as the current as large as possible it is necessary that this product be a maximum.

The upper family of curves in Figs. 7 and 8 shows the dependence of this product upon the direct current induction for Rings I and II respectively. The maxima of these occur near the inductions for which the direct current permeability is a maximum.

An increase in the direct current induction up to the point of maximum force sensitivity is accompanied by a considerable decrease in the alternating current permeability and hence a decrease in B_m in Eqn. 7 for a given alternating current. This decrease in B_m results in a decrease in the distortion of the pull arising from the double frequency term. Furthermore, it is of interest to note that the iron loss (Fig. 9) at the induction for which the force

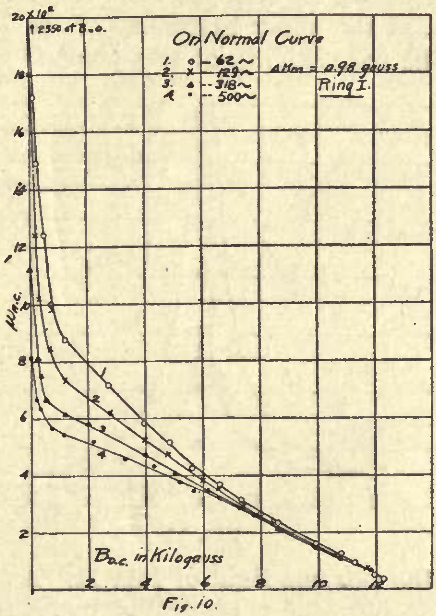

FIG. 10

function is a maximum is only a small fraction of its value with no superposed field, the alternating field remaining constant.

The curves of Fig. 9 show the variation of the total iron loss as a function of the direct current induction for an alternating field of 0.9 gauss at frequencies of 62, 128 and 500 cycles per second. Each curve shows that there is a decrease in the iron loss when the direct current induction is increased and the alternating current field kept at a constant maximum value. By means of his hysteresis tracer Ewing traced the small cycles produced by a given cyclic field when a constant field is superposed. He found that these small loops decrease in area as the direct current induction is increased. This decrease in area of these diminutive loops results in a decreased iron loss. The diminution in area of the loops is to be associated with the decrease in permeability resulting from the restraint imposed upon the elementary magnets by the constant field. A decrease in permeability would cause a decrease in the eddy current loss also.

The curves of Fig. 10 were taken with an alternating field of 0.98 gauss for different frequencies. The variation of permeability with frequency appears to be greatest with no superposed field and nearly to disappear when the direct current field is increased to a point where saturation occurs.

Fig. 11 shows the variation of the permeability with the direct current field for points on the hysteresis loop of Fig. 3. To trace this curve, start at the upper extremity of the loop with the field a maximum. The permeability at this point is given by the point

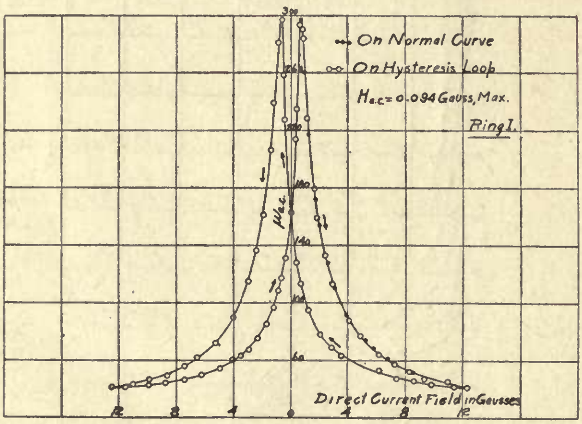

FIG. 11

on the curve at the extreme right of Fig. 11. As the direct current field is reduced to zero, the permeability increases along the lower of the curve to the right of the axis. As the field is increased in the opposite direction the permeability continues to increase and reaches a sharp maximum from which it falls rapidly, finally reaching the point on the curve at the extreme lower left of Fig. 11 as the field is increased to a maximum in this direction. The returning branch of the permeability curve starting from this point corresponds to the remaining half of the hysteresis loop starting at the lower extremity of the loop.

For comparison, permeabilities on the normal curve are plotted on the same graph. For the demagnetized specimen ($H_{d.c.} = 0$) the permeability reaches a value of 386. With increasing fields the permeability diminishes in such a way that the curve coincides with a part of the curve for the ascending branch of the hysteresis loop. Residual magnetism has reduced the permeability from 386, the value on the normal curve, to 160.

Similar curves have been made with an alternating field as large as 0.9 gauss. In this case the effect of the residual magnetism was apparently neutralized by the demagnetizing action of the alternating current field. The two maxima of the permeability curves coalesce on the permeability axis, and the permeability curve for points on the normal curve intersects the axis at the same point. The ascending and descending branches of the permeability curve are brought nearer together than they are in Fig. 11, but do not coincide.

In Fig. 12, curve 1 gives the variation of the permeability with the direct current induction for points on the hysteresis loop, the direction of tracing the permeability loop being indicated by the

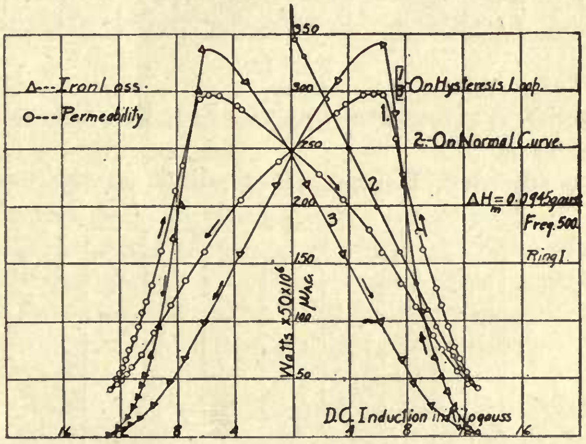

FIG. 12

direction of the arrow. In curve 2 the permeability for the normal curve is replotted. From these curves and Fig. 11, it appears that the permeability is not a simple function either of the superposed field or induction, but is affected by the previous magnetic history. Curve 3, of Fig. 12 represents the dependence of the iron loss upon the direct current induction for points on the hysteresis loop. The form of the curve is similar to that of the permeability curve, which is further evidence that the change in iron loss is due to the change in permeability. A similar set of curves for Ring II is given in Fig. 13. A curve of iron loss for the normal magnetization curve has been added to this group.

Oscillograms were made of current through the voltage across the winding of Ring I with the ring first demagnetized and then with progressively larger currents in the direct current winding. With no superposed direct current, the current wave showed the characteristic distortion due to hysteresis. As the direct current was increased the distortion decreased and, when the direct cur-

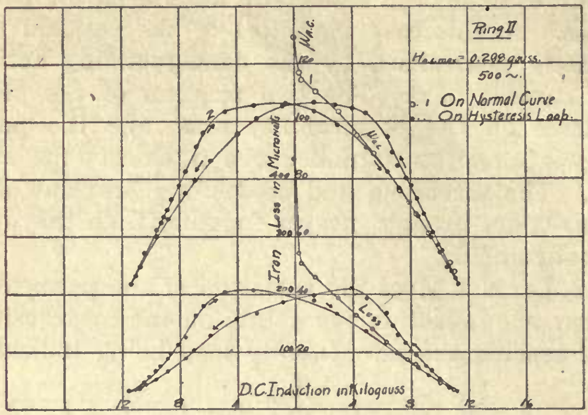

FIG. 13

rent required for saturation was reached, the distortion had entirely disappeared.

The force function, $B_{d.c.} \times \omega_{a.c.}$, for points on the normal curve

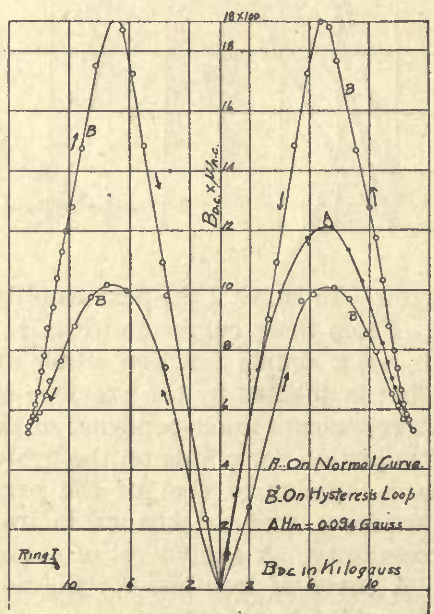

FIG. 14

and upon the hysteresis loop have been plotted in Fig. 14 against the direct current induction. From curve B it is seen that this force function increases to a maximum as the induction decreases from the maximum value to the remanent value; and that this product decreases to zero as the induction becomes zero. As the induction increases to a maximum in the opposite direction this product follows the lower curve to the left of the axis of Fig. 14, passing through a second maximum of roughly half the value of the first maximum. The maximum value attained by this product on the normal curve is intermediate in value between the maximum reached on either branch of the hysteresis loop.

SUMMARY

(1). The ordinary hysteresis loss for very slow cycles and moderate inductions obeys the parabolic law for all of the specimens studied.

(2). The total iron loss for rapid cycles is a parabolic function of the alternating current induction, when there is no superposed direct field.

(3). When there is no superposed constant field, the major part of the apparent change of permeability with frequency is due to the action of eddy currents.

(4). For a given value of alternating field the permeability and the iron loss each decreases as the direct current magnetization is increased from the demagnetized state; and when the iron is carried slowly through a hysteresis cycle, the permeability and energy loss pass through cycles. Each quantity reaches a maximum when the value of the direct current induction is equal to the remanent value.

(5). For small values of alternating field, the force function $B_{d.c.} x \mu_{a.c.}$ passes through two unequal maxima, on each branch of the hysteresis loop. It reaches on the normal magnetization curve a maximum value intermediate between the maxima on the hysteresis loop.

(6). The iron loss at the point at which the force function is a maximum is relatively small.

(7). For the three specimens on which observations have been made it is found that the magnitude of the effect of the superposed constant magnetic field is a function of the permeability of the specimen. The lower the permeability the smaller is the effect of the superposed field.

In conclusion, I wish to thank Professor Alpheus W. Smith for many helpful suggestions during the progress of this work. I desire to acknowledge also my indebtedness to Dr. H. D. Arnold of the Western Electric Company who kindly supplied me with the dust cores.

AUTOBIOGRAPHY

I, Alva Wellington Smith, was born in Fayette, Ohio, August 26, 1885. I received my preparatory education at the Fayette Normal and at the Michigan State Normal; my undergraduate education at the Ohio State University, from which I received the degree of Bachelor of Arts in 1912 and the degree of Master of Arts in 1914. I served on the corps of research engineers of the Western Electric Company during the year 1914-1915. I was an asistant in physics at the Ohio State University from 1912 to 1914 and an instructor at the same institution from 1915 to 1921. My post graduate education was received at the University of Chicago and at the Ohio State University from which I received the degree of Doctor of Philosophy in 1921.